L15 — Control Systems and Engineering Applications

Engineering Mathematics I

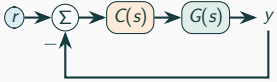
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Road map

- Control Theory Fundamentals
- Stability Analysis
- First- and Second-Order Responses
- Frequency Response
- Practical Engineering Applications



From system analysis to feedback control

Lecture 14 studied the response of a prescribed input:

$$Y(s) = G(s)U(s).$$

In control, the output is measured and compared with a desired value:

$$e(t) = r(t) - y(t), \qquad u(t) = \mathcal{C}\{e(t)\}.$$

Open loop

The input does not react to the output. Model errors and disturbances pass directly to the result.

Closed loop

The error changes the input. Feedback can reduce error, but it can also create unstable dynamics.

Control Theory Fundamentals

Two complementary model descriptions

State space

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}, \qquad \mathbf{y} = C\mathbf{x} + D\mathbf{u}.$$

Use it when internal variables or several inputs and outputs matter. The eigenvalues of A describe natural modes.

Transfer function

$$G(s) = \frac{Y(s)}{U(s)}.$$

Use it for a selected zero-state input–output channel. Poles and zeros make frequency and feedback analysis convenient.

Keep the distinction

A transfer function assumes zero initial conditions. State space retains the internal state and its initial value.

Example: one physical model, two descriptions

For a mass–spring–damper system,

$$m\ddot{x} + c\dot{x} + kx = u(t).$$

Choose $x_1 = x$ and $x_2 = \dot{x}$. Then

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -c/m \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1/m \end{bmatrix} u, \qquad y = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}.$$

With $\mathbf{x}(0) = 0$,

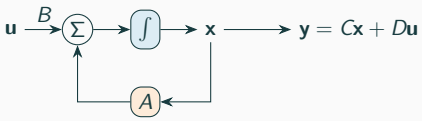
$$G(s) = \frac{X(s)}{U(s)} = \frac{1}{ms^2 + cs + k}.$$

Reading the model

The two state variables store the position and velocity. The denominator predicts the two natural modes.

State space makes the signal flow explicit

$$\text{State space: } \begin{cases} \dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \\ \mathbf{y} = C\mathbf{x} + D\mathbf{u} \end{cases}$$



- **x** is the minimum information that determines future motion once the future input is known.
- The matrix A governs the unforced internal dynamics.
- This form handles multiple inputs and outputs without changing the basic idea.

From state space to a transfer function

With $\mathbf{x}(0) = 0$, the Laplace transform gives

$$(sI - A)\mathbf{X} = B\mathbf{U}, \qquad G(s) = \frac{\mathbf{Y}}{\mathbf{U}} = C(sI - A)^{-1}B + D.$$

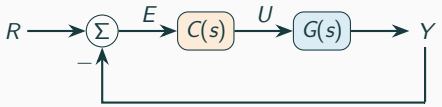
What is preserved

The transfer function records the selected input–output behavior, including its poles and zeros.

What is hidden

Internal coordinates and nonzero initial conditions are no longer visible. Different state realizations can have the same $G(s)$.

Unity negative feedback



$$E = R - Y, \qquad U = C(s)E, \qquad Y = G(s)U.$$

Define the loop transfer function $L(s) = C(s)G(s)$. It is the quantity used in both pole and Bode analysis.

Deriving the closed-loop transfer function

Substitute the loop equations one step at a time:

$$Y = GCE = GC(R - Y).$$

Rearranging gives

$$Y(1 + GC) = GC\,R, \qquad \boxed{T(s) = \frac{Y}{R} = \frac{L(s)}{1 + L(s)}}.$$

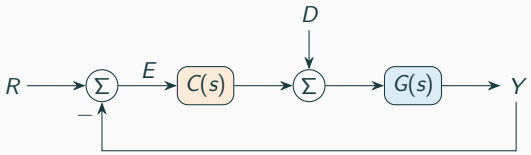
The denominator is the key

The equation $1 + L(s) = 0$ determines the closed-loop poles. Feedback therefore changes the natural modes, not only the output amplitude.

Checkpoint

Before discussing tracking, overshoot, or disturbance rejection, first ask: where are the closed-loop poles?

A disturbance uses the same denominator



Let a plant-input disturbance D enter after the controller. Then

$$Y = G\big(C(R - Y) + D\big).$$

Hence

$$\frac{Y}{R} = \frac{L}{1+L} = T(s), \qquad \frac{Y}{D} = \frac{G}{1+L} = G(s)S(s), \qquad S(s) = \frac{1}{1+L(s)}.$$

Interpretation

Large loop gain at the frequencies of interest can reduce both tracking error and the effect of this disturbance. It does not remove the need to check stability and noise.

Sensitivity separates tracking and disturbance rejection

Let $L(s) = C(s)G(s)$. Then

$$S(s) = \frac{1}{1 + L(s)}, \qquad T(s) = \frac{L(s)}{1 + L(s)}, \qquad S + T = 1.$$

Low frequency

Large $|L|$ makes $|S|$ small: improved tracking and rejection of plant-input disturbances.

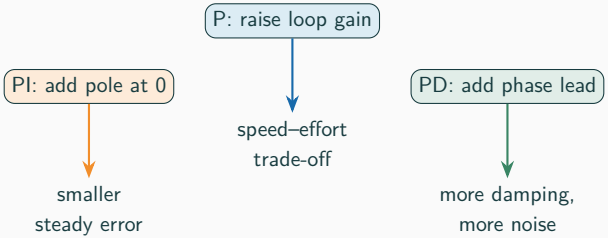
High frequency

Small $|T|$ limits measurement-noise transmission and neglected fast dynamics.

Stability must be established before either interpretation is valid.

P, PI, and PD actions alter different parts of the loop

Action	$C(s)$	Main benefit	Main caution
P	K_p	faster response	finite steady error
PI	$K_p + K_i/s$	removes constant error	reduced margin, windup
PD	$K_p + K_d s$	adds damping	amplifies noise



No single action improves every performance measure simultaneously.

Stability Analysis

Poles determine natural modes

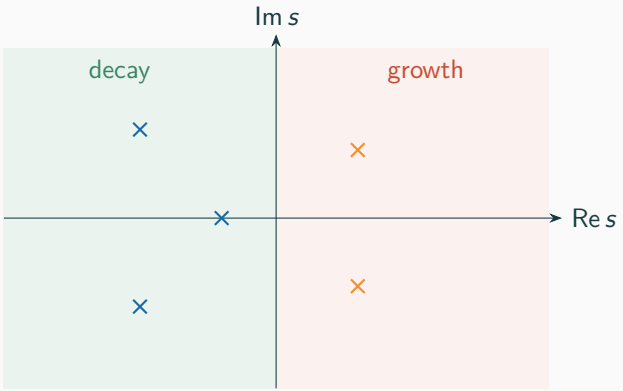
For simple poles p_k, the impulse response contains

$$h(t) = \sum_k r_k \mathrm{e}^{p_k t} u(t).$$

- $\operatorname{Re} p_k < 0$: decaying mode;
- $\operatorname{Re} p_k = 0$: persistent mode requiring additional care;
- $\operatorname{Re} p_k > 0$: growing mode.

A finite-dimensional continuous-time closed loop is asymptotically stable when every closed-loop pole lies in the open left half-plane.

The pole map makes stability visible



Distance to the imaginary axis controls decay rate; imaginary part controls oscillation frequency.

The characteristic equation is the stability gateway

For unity negative feedback,

$$T(s) = \frac{C(s)G(s)}{1 + C(s)G(s)}, \qquad 1 + C(s)G(s) = 0.$$

$$\text{model} \longrightarrow L(s) \longrightarrow \text{characteristic polynomial} \longrightarrow \text{closed-loop poles.}$$

Interpretation

Feedback does not merely scale the output; it changes the system's natural modes.

Worked example: gain moves the closed-loop poles

Let

$$G(s) = \frac{1}{s(s+2)}, \qquad C(s) = K.$$

Then

$$T(s) = \frac{K}{s^2 + 2s + K}.$$

Gain	Closed-loop poles	Response shape	Student reading
$K = 0.25$	$-1.866,\ -0.134$	slow, monotone	pole near 0 gives a long tail
$K = 1$	$-1,\ -1$	critical	fastest response without ringing
$K = 4$	$-1 \pm 1.732\mathrm{i}$	oscillatory	same decay rate, more ringing

Lesson

Increasing gain is not a universal improvement. It changes pole locations and therefore changes both speed and damping.

Zeros shape transmission but do not alone determine stability

$$G(s) = K\frac{\prod_j (s - z_j)}{\prod_k (s - p_k)}.$$

Poles set natural modes; zeros suppress or reshape particular input components.

Left-half-plane zero

Usually compatible with a minimum-phase response.

Right-half-plane zero

Can produce an inverse initial response and limit achievable bandwidth.

Exact cancellation can hide an unstable internal mode

The cancellation

$$\frac{s-1}{(s-1)(s+2)} = \frac{1}{s+2}$$

appears stable, yet a nonminimal realization can retain the internal mode e^{t}.

Practical rule

Do not cancel unstable poles in a design argument. Inspect the closed-loop state matrix and the realization.

The phase plane turns a second-order model into geometry

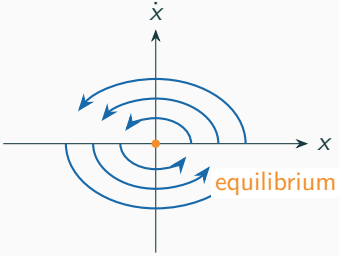
For an autonomous model

$$\ddot{x} + a\dot{x} + bx = 0, \qquad x_1 = x, \quad x_2 = \dot{x},$$

the state satisfies

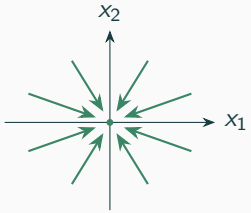
$$\dot{\mathbf{x}} = A\mathbf{x}, \qquad A = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix}.$$

- a point is an instantaneous state $(x, \dot{x})$;
- an arrow is the local state velocity;
- a trajectory is one complete time history.

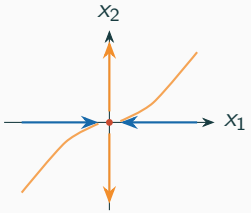


Real eigenvalues distinguish nodes from saddles

Let λ_1, λ_2 be the eigenvalues of a planar linear system.



stable node: $\lambda_1, \lambda_2 < 0$



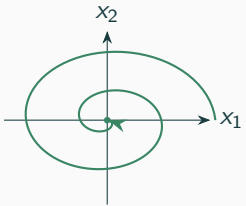
saddle: $\lambda_1 \lambda_2 < 0$

Engineering reading

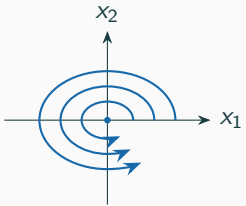
A stable node absorbs nearby disturbances; a saddle has at least one growing direction.

Complex eigenvalues distinguish spirals from centers

For $\lambda = \alpha \pm \mathrm{i}\beta$, the real part controls radial growth or decay, while $|\beta|$ controls rotation.



stable spiral: $\alpha < 0$



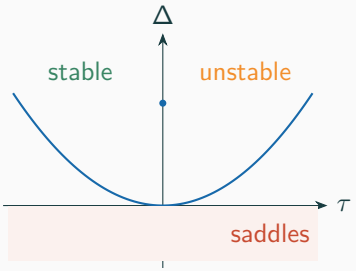
center: $\alpha = 0$

A center is not asymptotically stable: trajectories do not converge to the equilibrium.

Trace and determinant summarize every planar linear system

For $A \in \mathbb{R}^{2\times 2}$ define

$$\tau = \operatorname{tr} A, \qquad \Delta = \det A, \qquad \lambda^2 - \tau\lambda + \Delta = 0.$$



- $\Delta < 0$: saddle;
- $\Delta > 0$, $\tau < 0$: stable;
- $\Delta > 0$, $\tau > 0$: unstable;
- $\tau = 0$, $\Delta > 0$: center;
- $\tau^2 - 4\Delta$ selects node or spiral.

Nonlinear stability begins with local linearization

For

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}), \qquad \mathbf{f}(\mathbf{x}_e) = \mathbf{0},$$

write $\mathbf{x} = \mathbf{x}_e + \boldsymbol{\eta}$. Then

$$\dot{\boldsymbol{\eta}} = D\mathbf{f}(\mathbf{x}_e)\boldsymbol{\eta} + O(\|\boldsymbol{\eta}\|^2) = A_e\boldsymbol{\eta} + O(\|\boldsymbol{\eta}\|^2).$$

Decisive case

If no eigenvalue of A_e lies on the imaginary axis, the linearized type determines the local nonlinear behavior.

Inconclusive case

A zero-real-part eigenvalue requires nonlinear analysis; linearization alone cannot certify stability.

Worked Example: the pendulum has two different equilibria

With angle measured from the downward position,

$$\ddot{\theta} + \omega_0^2 \sin\theta = 0, \qquad \omega_0^2 = \frac{g}{\ell}.$$

For $\mathbf{x} = (\theta, \dot{\theta})$,

$$A_e = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 \cos\theta_e & 0 \end{bmatrix}.$$

Equilibrium	θ_e	eigenvalues	local phase portrait
hanging	0	$\pm i\omega_0$	center in the ideal model
upright	π	$\pm\omega_0$	saddle, hence unstable

Damping turns the ideal center into a stable spiral; feedback is required to stabilize the upright saddle.

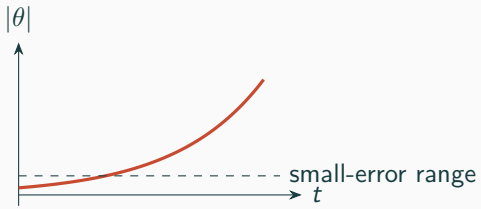
The pendulum's open-loop mode grows from a small error

Linearization about the upright equilibrium gives

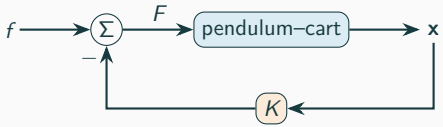
$$\dot{\mathbf{x}} = A\mathbf{x} + BF, \qquad \mathbf{x} = [x, \dot{x}, \theta, \dot{\theta}]^T.$$

A right-half-plane eigenvalue is obtained when $F = 0$.

A small angular error excites the unstable mode; growth is a model prediction, not a solver artifact.



State feedback moves the modes of the linearized model



With

$$F = f - K\mathbf{x}, \qquad \dot{\mathbf{x}} = (A - BK)\mathbf{x} + Bf,$$

the workflow is

1. identify unstable modes of A;
2. form $A - BK$ and verify its eigenvalues;
3. simulate recovery from representative disturbances;
4. check force limits and the linearization range.

Systematic pole placement is outside the present course.

Feedback changes the upright local phase portrait

Let $\phi = \theta - \pi$. Near the upright equilibrium,

$$\ddot{\phi} = \omega_0^2 \phi + u, \qquad u = -k_p \phi - k_d \dot{\phi}.$$

Hence

$$\ddot{\phi} + k_d \dot{\phi} + (k_p - \omega_0^2)\phi = 0.$$

Open loop

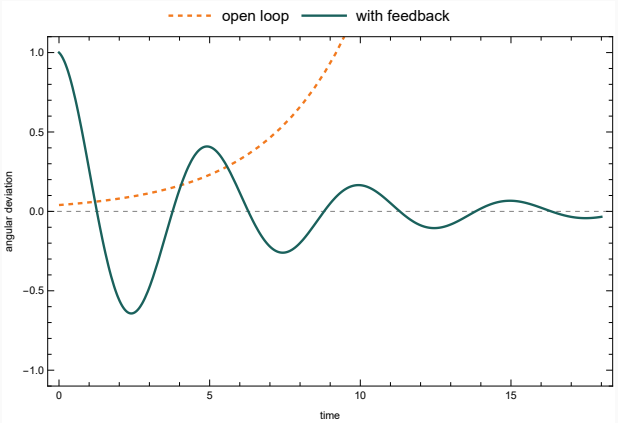
$k_p = k_d = 0$: one positive eigenvalue and a saddle portrait.

Locally stable feedback

$k_d > 0$ and $k_p > \omega_0^2$: both characteristic coefficients are positive.

This is a local conclusion; actuator limits and the nonlinear angle range still matter.

Feedback can turn growth into decay



Open-loop deviation grows, whereas suitable feedback returns the pendulum toward equilibrium.

First- and Second-Order Responses

A first-order closed loop has one dominant time scale

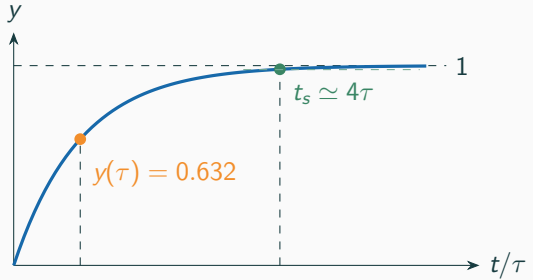
For a unit step and $T(s) = 1/(\tau s + 1)$,

$$Y(s) = \frac{1}{s(\tau s + 1)} \quad \Longrightarrow \quad y(t) = 1 - \mathrm{e}^{-t/\tau}.$$

Hence

$$y(\tau) = 0.632, \qquad t_s(2\%) \approx 4\tau.$$

The time constant is both a model parameter and a directly observable response feature.



Worked example: proportional temperature regulation

A thermal plant is $G(s) = 2/(5s+1)$ with $C(s) = K$:

$$T(s) = \frac{2K}{5s + 1 + 2K}.$$

For $K = 2$,

$$T(0) = \frac{4}{5}, \qquad \tau_{cl} = \frac{5}{1+2K} = 1 \text{ s},$$

so

$$y(t) = \frac{4}{5}(1 - \mathrm{e}^{-t})$$

for a unit step. Feedback accelerates the response but leaves $e_{ss} = 1/5$.

Steady-state error follows from the final-value theorem

$$E(s) = \frac{R(s)}{1 + L(s)}, \qquad e_{ss} = \lim_{s \to 0} sE(s),$$

provided every pole of $sE(s)$ lies in the open left half-plane.

Loop type	Step input	Ramp input
Type 0	finite error	unbounded error
Type 1	zero error	finite error

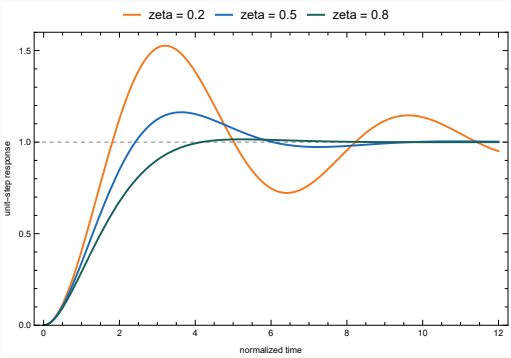
Integral action raises system type but can reduce stability margin.

The standard second-order form separates speed and damping

$$T(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2},$$

$$p_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}.$$

- ω_n mainly sets the time scale;
- ζ mainly sets oscillation and overshoot;
- $0 < \zeta < 1$ underdamped, $\zeta = 1$ critical, $\zeta > 1$ overdamped.



Lower damping gives faster initial motion but larger overshoot and longer ringing.

Useful underdamped response estimates

For $0 < \zeta < 1$, let $\omega_d = \omega_n\sqrt{1-\zeta^2}$:

$$M_p = \mathrm{e}^{-\pi\zeta/\sqrt{1-\zeta^2}}, \qquad t_p = \frac{\pi}{\omega_d}, \qquad t_s \approx \frac{4}{\zeta\omega_n}.$$

These formulas estimate the dominant second-order transient; a higher-order response must still be checked directly.

Worked example: poles from transient specifications

Require $M_p \le 10\%$ and $t_s \le 2$ s:

$$\zeta \gtrsim 0.59, \qquad \zeta\omega_n \ge 2.$$

Choose $\zeta = 0.7$ and $\omega_n = 3$:

$$p_{1,2} = -2.1 \pm 2.14\mathrm{i}, \quad M_p \approx 4.6\%, \quad t_s \approx 1.90\ \mathrm{s}.$$

Both requirements are satisfied, but bandwidth and actuator effort remain to be checked.

Dominant poles explain many higher-order responses

If one pole pair lies much closer to the imaginary axis than every other pole, it often dominates the transient response.

Useful approximation

Estimate settling time and oscillation from the dominant pair, then verify the full response.

When it fails

Nearby zeros, lightly damped neglected modes, delay, or near pole–zero cancellation can reshape the response strongly.

Response measures form a design trade-off

Quantity	Main influence	Typical tension
Rise time	bandwidth	noise sensitivity
Overshoot	damping ratio	response speed
Settling time	dominant real part	actuator effort
Steady error	low-frequency gain	stability margin

Design reading

No single number describes a good control system. A specification must state which response features matter for the application.

Frequency Response

Sinusoidal steady state samples the transfer function

For stable $G(s)$ and $u(t) = A\cos\omega t$,

$$y_{ss}(t) = A|G(\mathrm{i}\omega)|\cos(\omega t + \arg G(\mathrm{i}\omega))\,.$$

Magnitude

$|G(\mathrm{i}\omega)|$ says whether a sinusoid at frequency ω is amplified or attenuated.

Phase

$\arg G(\mathrm{i}\omega)$ says how far the output leads or lags the input. Excess lag can turn negative feedback into positive feedback.

Bode plots separate gain and phase over decades

$$M(\omega) = 20 \log_{10} |G(\mathrm{i}\omega)| \ \mathrm{dB}, \qquad \phi(\omega) = \arg G(\mathrm{i}\omega).$$

Products become sums:

$$G = K \prod_j G_j \Longrightarrow M = 20 \log_{10} |K| + \sum_j M_j, \qquad \phi = \arg K + \sum_j \phi_j.$$

This makes pole and zero contributions reusable.

Reusable one-pole pattern

For $G(s) = 1/(1 + s/\omega_p)$,

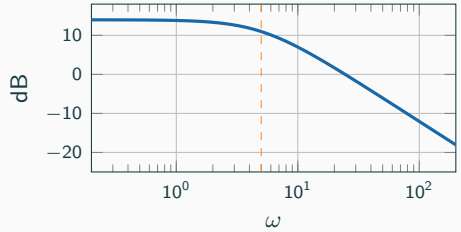
$$|G(\mathrm{i}\omega)| = \frac{1}{\sqrt{1 + (\omega/\omega_p)^2}}, \qquad \phi = -\tan^{-1}(\omega/\omega_p).$$

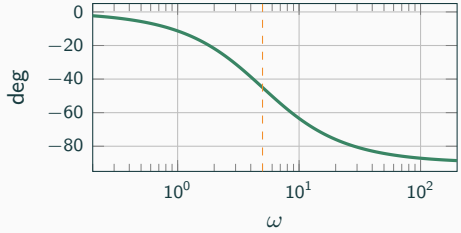
- If $\omega \ll \omega_p$, magnitude is approximately 0 dB and phase approximately 0°.
- At $\omega = \omega_p$, magnitude is -3 dB and phase is -45°.
- If $\omega \gg \omega_p$, magnitude approaches a -20 dB/decade slope and phase approaches -90°.

Worked example: a complete first-order Bode plot

For $G(s) = 5/(1 + 0.2s)$,

$$M = 20\log_{10} 5 - 10\log_{10}(1 + 0.04\omega^2), \qquad \phi = -\tan^{-1}(0.2\omega).$$





The corner is 5 rad/s, where the magnitude is 3 dB below its low-frequency value and phase is $-45°$.

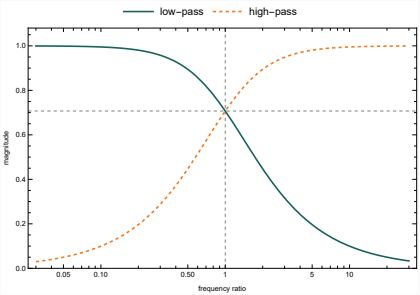
Low-pass and high-pass responses emphasize different bands

Low-pass behavior

Low-frequency signals pass with relatively little attenuation, while fast fluctuations are suppressed. This is useful when the desired signal is slow and measurement noise is fast.

High-pass behavior

Slow variation is attenuated while rapid change passes. This can reveal fast events, but it does not preserve a steady reference level.



The corner frequency marks the transition between passed and attenuated content.

Gain and phase margins measure robustness

At the gain-crossover frequency, $|L(\mathrm{i}\omega_{gc})| = 1$:

$$\mathrm{PM} = 180^\circ + \arg L(\mathrm{i}\omega_{gc}).$$

At a phase-crossover frequency, $\arg L(\mathrm{i}\omega_{pc}) = -180^\circ$:

$$\mathrm{GM}_{\mathrm{dB}} = -20 \log_{10} |L(\mathrm{i}\omega_{pc})|.$$

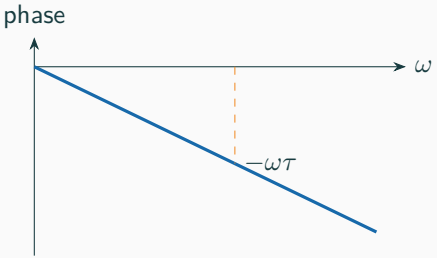
Worked reading

If $\arg L(\mathrm{i}4) = -135^\circ$ and $20 \log_{10} |L(\mathrm{i}9)| = -8$ dB, then $\mathrm{PM} = 45^\circ$ and $\mathrm{GM} = 8$ dB.

Positive margins give room for uncertainty. They do not guarantee robustness against every unmodeled effect.

Delay consumes phase margin

$$G_d(s) = \mathrm{e}^{-s\tau}, \qquad |G_d(\mathrm{i}\omega)| = 1, \qquad \arg G_d(\mathrm{i}\omega) = -\omega\tau.$$



Engineering reading

Delay does not change magnitude, but it adds lag that grows with frequency. A fast loop is therefore especially vulnerable to delay.

Bandwidth connects speed, filtering, and uncertainty

A wider closed-loop bandwidth usually improves tracking speed. It also passes more measurement noise, demands faster actuator motion, and exposes neglected high-frequency dynamics.

Low frequency

Prefer large $|L|$: small $|S|$ improves tracking and attenuates low-frequency plant-input disturbances.

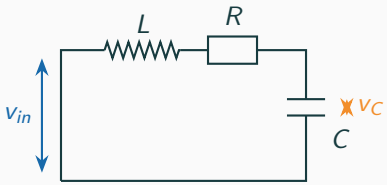
High frequency

Prefer small $|T|$: measurement noise and unmodeled fast dynamics transmit less strongly to the output.

The design problem is to choose useful bandwidth, not the largest possible bandwidth.

Practical Engineering Applications

Case study I: an RLC circuit is a second-order plant



$$LC\ddot{v}_C + RC\dot{v}_C + v_C = v_{in},$$

$$G(s) = \frac{1}{LCs^2 + RCs + 1}.$$

$$\omega_n = \frac{1}{\sqrt{LC}}, \qquad \zeta = \frac{R}{2}\sqrt{\frac{C}{L}}.$$

- increasing R raises damping and reduces resonance;
- increasing L or C lowers the natural frequency;
- a measured step response can estimate ζ and ω_n;
- feedback may reshape performance, but voltage and current limits remain physical.

Bridge to control

The same pole and response language applies to mechanical, electrical, and thermal plants.

Case study II: regenerative cutting introduces delay

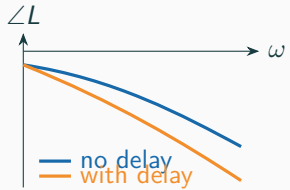
A simplified lathe model contains the previous-revolution depth:

$$m\ddot{x} + c\dot{x} + kx = F(x(t) - x(t-\tau)) .$$

The delay contributes $\mathrm{e}^{-s\tau}$, so the characteristic equation is transcendental rather than polynomial.

Current cutting force depends on a surface created one revolution earlier; delay can reinforce vibration and generate chatter.

Delay can destroy phase margin



- magnitude may appear unchanged;
- phase crossing occurs earlier;
- phase margin decreases;
- chatter risk increases.

$$\arg \mathrm{e}^{-\mathrm{i}\omega\tau} = -\omega\tau.$$

Case study III: nonlinear pendulum–cart dynamics

A representative nonlinear model is

$$(M+m)\ddot{x} + ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F,$$

$$l\ddot{\theta} + \ddot{x}\cos\theta - g\sin\theta = 0.$$

Products such as $\ddot{\theta}\cos\theta$ make the model nonlinear; the upright equilibrium is the operating point of interest.

Near the upright equilibrium use

$$\sin\theta \approx \theta, \qquad \cos\theta \approx 1, \qquad \dot{\theta}^2\theta \approx 0.$$

Then

$$\dot{\mathbf{x}} = A\mathbf{x} + BF, \qquad \mathbf{x} = [x, \dot{x}, \theta, \dot{\theta}]^T.$$

Validity

Closed-loop conclusions are local; large-angle motion, saturation, and sensor limits require the nonlinear model.

Disturbance recovery is a separate engineering test

For a disturbance entering the force channel, a state-feedback model gives

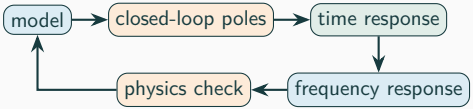
$$\mathbf{X}(s) = (sI - A + BK)^{-1} B\, D(s).$$

A useful test records more than whether the trajectory eventually decays:

1. peak angle after the disturbance;
2. recovery time;
3. peak control force or voltage;
4. whether the state stays inside the linearization range.

A design that is stable on paper may fail this test because of actuator limits or an invalid local model.

Verification closes the engineering loop



- Check actuator force, voltage, current, and saturation.
- Test representative disturbances and measurement noise.
- Check whether the state remains within the model's validity range.
- Revise the model or controller when any check fails.

Common interpretation errors

- Declaring stability from zeros instead of closed-loop poles.
- Treating high gain as universally beneficial.
- Applying the final-value theorem without checking its pole condition.
- Reporting overshoot without specifying the input and normalization.
- Reading Bode magnitude while ignoring phase and delay.
- Relying on cancellation of an unstable pole.
- Applying a local linear model far from its operating point.

A reliable habit

State the model, operating conditions, and performance measure before drawing a control conclusion.

Method-selection guide

Question	Primary tool
What variables store the system memory?	state space
What is the zero-state input–output relation?	transfer function
Is the closed loop stable?	closed-loop poles
Does it meet speed, overshoot, and error targets?	step response
How does it amplify or lag sinusoids?	Bode response
How much uncertainty or delay can it tolerate?	gain and phase margins
Can the physical system realize the result?	constraints and validation

Summary

1. Build $L(s) = C(s)G(s)$, then obtain $T(s) = L/(1 + L)$.
2. Closed-loop poles determine stability and dominant transient behavior.
3. First- and second-order models connect poles to settling time, overshoot, and steady error.
4. Bode plots, bandwidth, and stability margins reveal the speed–robustness trade-off.
5. A correct design also respects delay, actuator limits, noise, and model validity.

Next $\gg$ Lecture 16 — Advanced Topics and Review